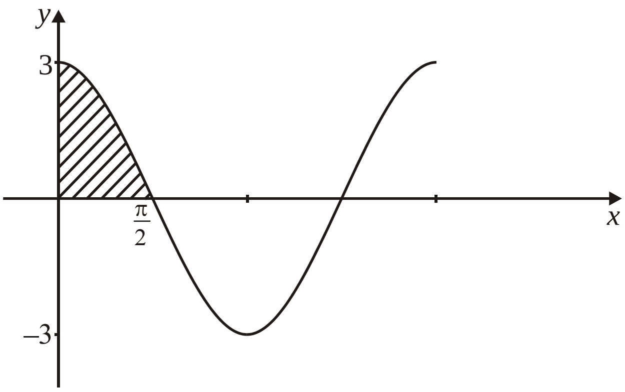
BANCO DE PROBLEMAS PREPARATORIOS
PARA EXAMEN I QUIMESTRE
2015 – 2016

1. The graph represents the function

$$f : x \mapsto p\cos x,\ p \in \mathbb{N}.$$



Find

(a) the value of p;

(b) the area of the shaded region.

(Total 4 marks)

2. Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

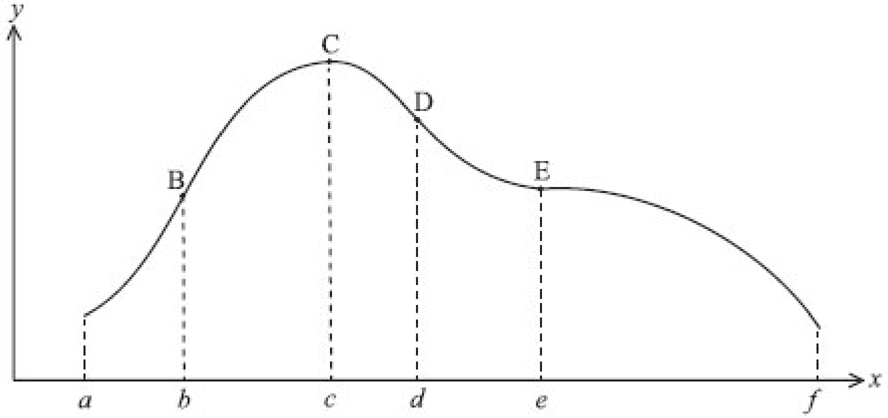
(a) Use the quotient rule to show that $g'(x) = \frac{1-2\ln x}{x^3}$.

(4)

(b) The graph of g has a maximum point at A. Find the x-coordinate of A.

(3)
(Total 7 marks)

3. The graph of a function g is given in the diagram below.



The gradient of the curve has its maximum value at point B and its minimum value at point D. The tangent is horizontal at points C and E.

(a) Complete the table below, by stating whether the first derivative g' is positive or negative, and whether the second derivative g'' is positive or negative.

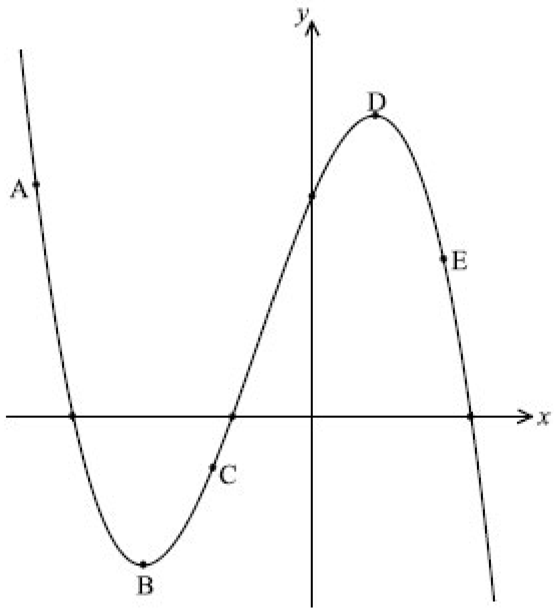
Interval	g'	g''
$a < x < b$		
$e < x < f$		

(b) Complete the table below by noting the points on the graph described by the following conditions.

Conditions	Point
$g'(x) = 0,\ g''(x) < 0$	
$g'(x) < 0,\ g''(x) = 0$	

(Total 6 marks)

4. The following diagram shows part of the curve of a function f. The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

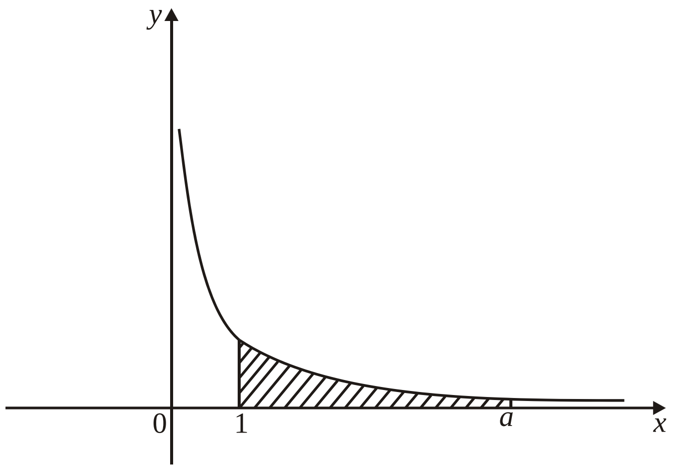
	A	B	E
$f'(x)$			

(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	C	E
$f''(x)$			

(Total 6 marks)

5. The diagram shows part of the graph of $y = \frac{1}{x}$. The area of the shaded region is 2 units.

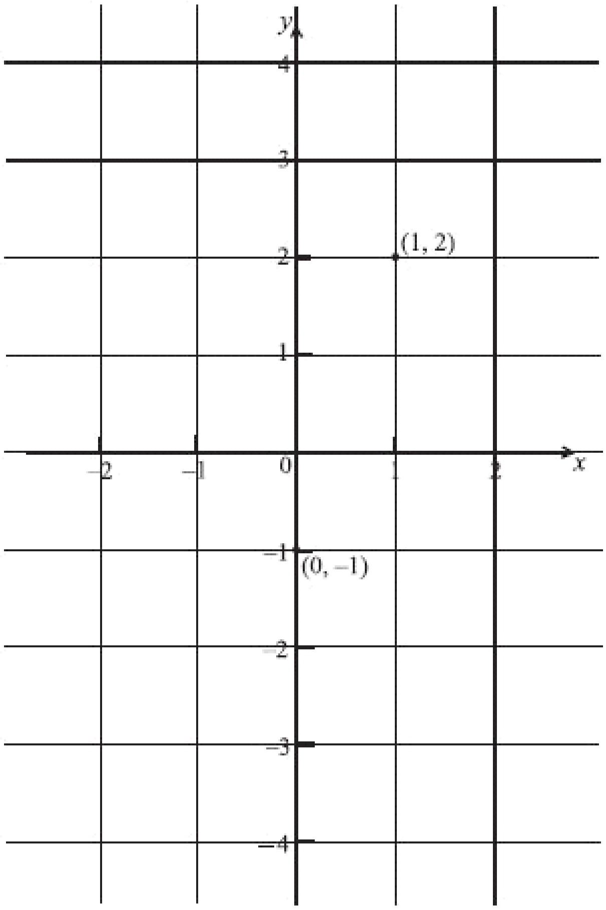


Find the exact value of a.

(Total 4 marks)

6. On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \le x < 0$		negative	positive
0	–1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \le 2$		positive	negative



(Total 6 marks)

7. Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

(Total 6 marks)

8. Let $f(x) = \dfrac{1}{1+x^2}$.

(a) Write down the equation of the horizontal asymptote of the graph of f.

(1)

(b) Find $f'(x)$.

(3)

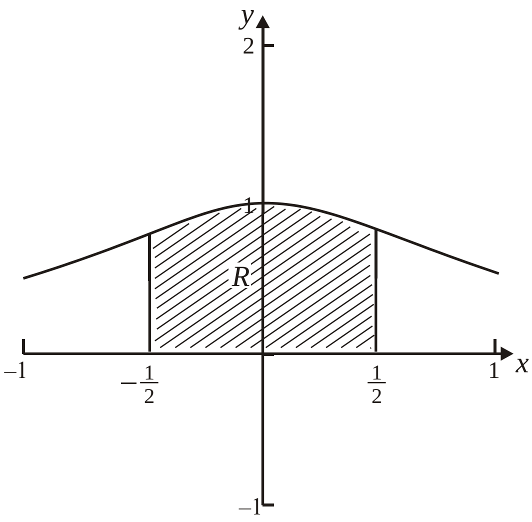
(c) The second derivative is given by $f''(x) = \dfrac{6x^2-2}{(1+x^2)^3}$.

Let A be the point on the curve of f where the gradient of the tangent is a maximum. Find the x-coordinate of A.

(4)

(d) Let R be the region under the graph of f, between $x = -\dfrac{1}{2}$ and $x = \dfrac{1}{2}$,

as shaded in the diagram below



Write down the definite integral which represents the area of R.

(2)
(Total 10 marks)

9. Let $f(x) = kx^4$. The point P(1, k) lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

(Total 6 marks)

10. Given $\int_3^k \frac{1}{x-2}\,dx = \ln 7$, find the value of k.

(Total 6 marks)

11. Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

(a) Find $f'(x)$.

(2)

(b) Find $g'(x)$.

(2)

(c) Let $h(x) = f(x) \times g(x)$. Find $h'(x)$.

(2)
(Total 6 marks)

12. The table below shows some values of two functions, f and g, and of their derivatives f' and g'.

x	1	2	3	4
$f(x)$	5	4	–1	3
$g(x)$	1	–2	2	–5
$f'(x)$	5	6	0	7
$g'(x)$	–6	–4	–3	4

Calculate the following.

(a) $\frac{\mathrm{d}}{\mathrm{d}x}(f(x) + g(x))$, when $x = 4$;

(b) $\int_1^3 (g'(x) + 6)\mathrm{d}x$.

(Total 6 marks)

13. Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

(a) Write down

(i) $f'(x)$;

(ii) $g'(x)$.

(2)

(b) Let $h(x) = \mathrm{e}^{-3x} \sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$.

(4)
(Total 6 marks)

14. It is given that $\int_1^3 f(x)\mathrm{d}x = 5$.

(a) Write down $\int_1^3 2f(x)\mathrm{d}x$.

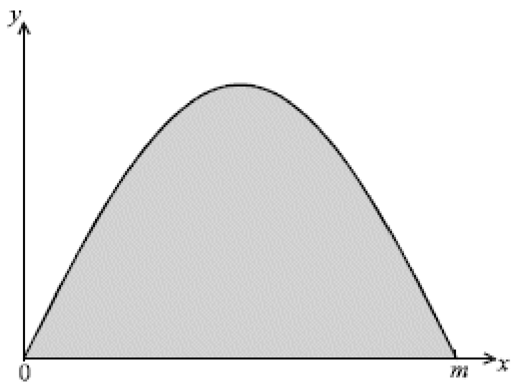
(b) Find the value of $\int_1^3 (3x^2 + f(x))\mathrm{d}x$.

(Total 6 marks)

15. The velocity v of a particle at time t is given by $v = \mathrm{e}^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

(Total 6 marks)

16. The diagram below shows part of the graph of $y = \sin 2x$. The shaded region is between $x = 0$ and $x = m$.



(a) Write down the period of this function.

(2)

(b) Hence or otherwise write down the value of m.

(2)

(c) Find the area of the shaded region.

(6)
(Total 10 marks)

17. (a) Find $\int_1^2 (3x^2 - 2)\,\mathrm{d}x$.

(4)

(b) Find $\int_0^1 2\mathrm{e}^{2x}\,\mathrm{d}x$.

(3)
(Total 7 marks)

18. Consider $f(x) = x^2 + \dfrac{p}{x}$, $x \neq 0$, where p is a constant.

(a) Find $f'(x)$.

(2)

(b) There is a minimum value of $f(x)$ when $x = -2$. Find the value of p.

(4)
(Total 6 marks)

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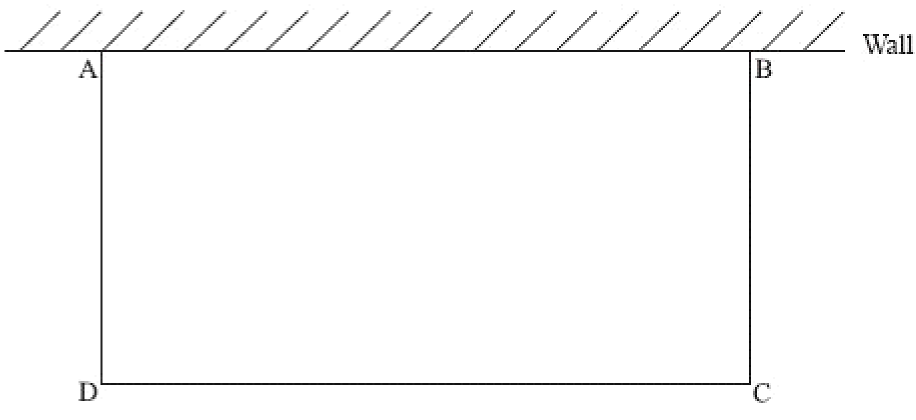
19. The following diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.



Find the width of the rectangle that gives its maximum area.

(Total 6 marks)

20. Let $f(x) = 3\cos 2x + \sin^2 x$.

(a) Show that $f'(x) = -5\sin 2x$.

(b) In the interval $\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$, one normal to the graph of f has equation $x = k$.

Find the value of k.

(Total 6 marks)

21. (a) Find $\int \frac{1}{2x+3}\mathrm{d}x$.

(2)

(b) Given that $\int_0^3 \frac{1}{2x+3}\mathrm{d}x = \ln\sqrt{P}$, find the value of P.

(4)
(Total 6 marks)

22. The acceleration, a m s^{-2}, of a particle at time t seconds is given by $a = 2t + \cos t$.

(a) Find the acceleration of the particle at $t = 0$.

(2)

(b) Find the velocity, v, at time t, given that the initial velocity of the particle is 2 m s^{-1}.

(5)

(c) Find $\int_0^3 v\,\mathrm{d}t$, giving your answer in the form $p - q\cos 3$.

(7)

(d) What information does the answer to part (c) give about the motion of the particle?

(2)
(Total 16 marks)

23. Let $\int_1^5 3f(x)\,\mathrm{d}x = 12$.

(a) Show that $\int_5^1 f(x)\,\mathrm{d}x = -4$.

(2)

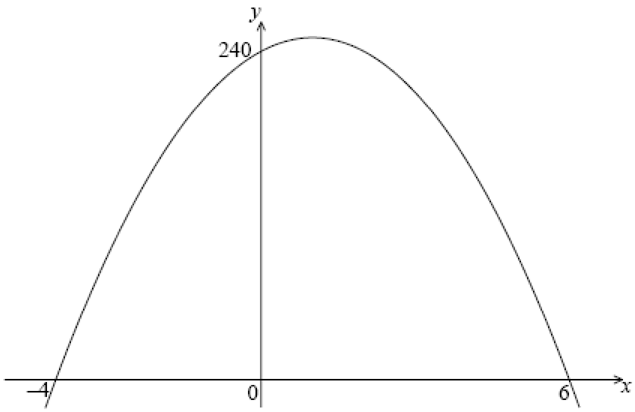
(b) Find the value of $\int_1^2 (x + f(x))\,\mathrm{d}x + \int_2^5 (x + f(x))\,\mathrm{d}x$.

(5)
(Total 7 marks)

24. A particle moves along a straight line so that its velocity, v m s^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

(Total 7 marks)

25. The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at (–4, 0) and (6, 0) and the y-intercept is at (0, 240).

(a) Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

(2)

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.

(4)

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

(2)

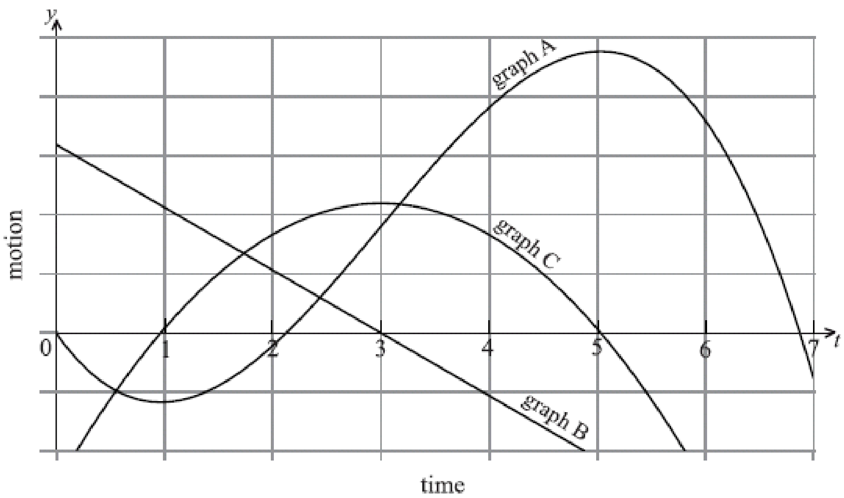
A particle moves along a straight line so that its velocity, v m s^{-1}, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

(7)
(Total 15 marks)

26. The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.



(a) Complete the following table by noting which graph A, B or C corresponds to each function.

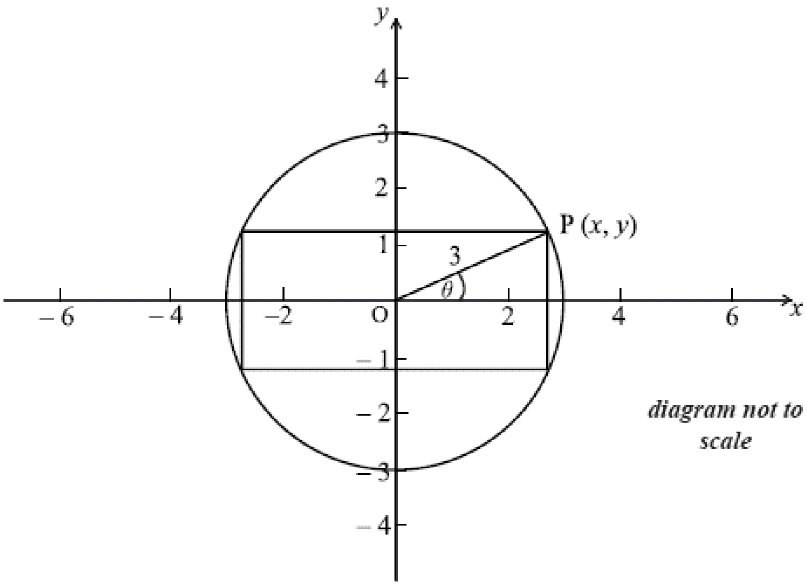
Function	Graph
displacement	
acceleration	

(4)

(b) Write down the value of t when the velocity is greatest.

(2)
(Total 6 marks)

27. A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x, y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

(a) Write down an expression in terms of θ for

(i) x;

(ii) y.

(2)

Let the area of the rectangle be A.

(b) Show that $A = 18 \sin 2\theta$.

(3)

(c) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$.

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

(8)
(Total 13 marks)

28. The graph of the function $y = f(x)$ passes through the point $\left(\frac{3}{2}, 4\right)$. The gradient function of f is given as $f'(x) = \sin(2x - 3)$. Find $f(x)$.

(Total 6 marks)